

Oases in the Desert: Three New Proposals

Ernest Ma

Physics Department, University of California, Riverside, USA

Abstract. Two new nontrivial U(1) gauge symmetries are proposed, one based on the particle content of the standard model and the other on that of its supersymmetric generalization. Each is an unexpected first example of its kind. A third new proposal is the successful derivation of a realistic Majorana neutrino mass matrix, based on the underlying symmetry A_4 and its radiative corrections.

1. Introduction

The title of the talk I gave at *Beyond the Desert 2002* was “New Unexpected Gauge Extensions of the Standard Model”. At the request of the editor of these Proceedings, I am including some very recent work on the prediction of the Majorana neutrino mass matrix, hence the present title.

I start with a brief review of the symmetries of the Standard Model and previous (trivial) gauge extensions. I then discuss two newly discovered nontrivial U(1) gauge extensions [1, 2], based on the particle content of the Standard Model and its supersymmetric generalization respectively. Finally I switch gear and present a new natural understanding of the Majorana neutrino mass matrix, which automatically gives the observed pattern of solar [3] and atmospheric [4] neutrino oscillations and a prediction of neutrinoless double beta decay at the level of 0.4 eV [5].

2. Anomaly Cancellation in the Standard Model

The gauge group of the Standard Model is $SU(3)_C \times SU(2)_L \times U(1)_Y$, under which each family of quarks and leptons transforms as follows:

$$(u, d)_L \sim (3, 2, 1/6), \quad u_R \sim (3, 1, 2/3), \quad d_R \sim (3, 1, -1/3); \quad (1)$$

$$(\nu, e)_L \sim (1, 2, -1/2), \quad e_R \sim (1, 1, -1). \quad (2)$$

The axial-vector triangle anomaly [6] is absent because [7]

$$[SU(3)]^2 Y : \left(\frac{1}{2}\right) \left[2 \left(\frac{1}{6}\right) - \frac{2}{3} - \left(-\frac{1}{3}\right) \right] = 0, \quad (3)$$

$$[SU(2)]^2 Y : \left(\frac{1}{2}\right) \left[3 \left(\frac{1}{6}\right) + \left(-\frac{1}{2}\right) \right] = 0, \quad (4)$$

$$Y^3 : 6 \left(\frac{1}{6}\right)^3 - 3 \left(\frac{2}{3}\right)^3 - 3 \left(-\frac{1}{3}\right)^3 + 2 \left(-\frac{1}{2}\right)^3 - (-1)^3 = 0. \quad (5)$$

Note the nontrivial cancellation between quarks and leptons in the last two equations. The original 1967 model of leptons by Weinberg [8] was thus anomalous.

3. Automatic Symmetries of the Standard Model

Given the minimal particle content of the Standard Model, there are four automatic global symmetries: baryon number B and the 3 lepton numbers L_e, L_μ, L_τ . Each is anomalous but the combination $a_B B + a_e L_e + a_\mu L_\mu + a_\tau L_\tau$ is not for the following cases.

(1) If there is no $N_R \sim (1, 1, 0)$, then the only solution is $a_B = 0$ and $(a_e, a_\mu, a_\tau) = (1, -1, 0)$, $(1, 0, -1)$, or $(0, 1, -1)$. This allows $L_i - L_j$ to be gauged [9].

(2) If only one family has an additional N_R , then the only solution is $a_B = 1$ and $(a_e, a_\mu, a_\tau) = (-3, 0, 0)$, $(0, -3, 0)$, or $(0, 0, -3)$. This allows $B - 3L_i$ to be gauged [10].

(3) If two N_R 's are added, then $a_B = 1$ and $a_e = 0, a_\mu + a_\tau = -3$, or $a_\mu = 0, a_e + a_\tau = -3$, or $a_\tau = 0, a_e + a_\mu = -3$ is a solution. This allows for example $B - (3/2)(L_\mu + L_\tau)$ to be gauged [11].

(4) If there are three N_R 's, then $a_B = 1$ and $a_e + a_\mu + a_\tau = -3$ is a solution. The well-known example of $a_e = a_\mu = a_\tau = -1$ allows $B - L$ to be gauged [12]. Another solution is $a_B = 0$ and $a_e + a_\mu + a_\tau = 0$. This means for example that $2L_e - L_\mu - L_\tau$ may be gauged.

4. Neutrino Mass and a New U(1) Gauge Symmetry

The addition of N_R allows the doublet neutrinos ν_i to acquire small Majorana masses via the famous seesaw mechanism. However, if N_R is replaced by a heavy fermion triplet

$$(\Sigma^+, \Sigma^0, \Sigma^-)_R \sim (1, 3, 0), \quad (6)$$

neutrinos get seesaw masses just as effectively [13, 14].

Since the addition of one N_R per family leads to the well-known U(1) gauge symmetry $B - L$, the same question may be raised as to the addition of one Σ_R triplet per family. It has recently been shown [1] that indeed such an $U(1)_X$ exists, under which

$$(u, d)_L \sim n_1, \quad u_R \sim n_2 = \frac{1}{4}(7n_1 - 3n_4), \quad d_R \sim n_3 = \frac{1}{4}(n_1 + 3n_4), \quad (7)$$

$$(\nu, e)_L \sim n_4, \quad e_R \sim n_5 = \frac{1}{4}(-9n_1 + 5n_4), \quad \Sigma_R \sim n_6 = \frac{1}{4}(3n_1 + n_4). \quad (8)$$

Note that this does not correspond to any combination of known quantum-number assignments, such as Q, Y, B or L .

To show that $U(1)_X$ has no anomalies, consider first

$$[SU(3)]^2 X : 2n_1 - n_2 - n_3 = 0, \quad \text{and} \quad Y^2 X : \quad (9)$$

$$6 \left(\frac{1}{6}\right)^2 n_1 - 3 \left(\frac{2}{3}\right)^2 n_2 - 3 \left(-\frac{1}{3}\right)^2 n_3 + 2 \left(-\frac{1}{2}\right)^2 n_4 - (-1)^2 n_5 = 0, \quad (10)$$

$$\text{and} \quad [SU(2)]^2 X : \left(\frac{1}{2}\right) (3n_1 + n_4) - (2)n_6 = 0. \quad (11)$$

These imply

$$n_3 = 2n_1 - n_2, \quad n_5 = -\frac{1}{2}n_1 - n_2 + \frac{1}{2}n_4, \quad n_6 = \frac{1}{4}(3n_1 + n_4). \quad (12)$$

Consider next YX^2 :

$$6\left(\frac{1}{6}\right)n_1^2 - 3\left(\frac{2}{3}\right)n_2^2 - 3\left(-\frac{1}{3}\right)n_3^2 + 2\left(-\frac{1}{2}\right)n_4^2 - (-1)n_5^2 = 0. \quad (13)$$

Using Eq. (12), this implies

$$\frac{1}{4}(3n_1 + n_4)(7n_1 - 4n_2 - 3n_4) = 0. \quad (14)$$

If $3n_1 + n_4 = 0$, then $n_6 = 0$ as well, and $U(1)_X$ is proportional to $U(1)_Y$, i.e. no new gauge symmetry has been discovered. On the other hand, if $7n_1 - 4n_2 - 3n_4 = 0$ is chosen instead, the solution given by Eqs. (7) and (8) is obtained. Nevertheless, there is still one more condition to be checked, i.e. the sum of the cubes of all $U(1)_X$ charges. Remarkably,

$$X^3 : 6n_1^3 - 3n_2^3 - 3n_3^3 + 2n_4^3 - n_5^3 - 3n_6^3 = 0 \quad (15)$$

automatically. Furthermore, the sum of all $U(1)_X$ charges themselves,

$$X : 6n_1 - 3n_2 - 3n_3 + 2n_4 - n_5 - 3n_6 = 0 \quad (16)$$

as well. Hence the mixed gravitational-gauge anomaly [15] is also absent automatically. These are highly nontrivial results.

The Higgs sector of this gauge extension requires two doublets, one with $U(1)_X$ charge $(9n_1 - n_4)/4$ to give mass to the charged leptons, and the other with charge $3(n_1 - n_4)/4$ for the other fermions.

In general, for the fermion multiplet $(1, 2p + 1, 0; n_6)_R$, there are 3 conditions to be satisfied, i.e. the analogs of Eqs. (11), (15) and (16):

$$\frac{1}{2}(3n_1 + n_4) = \frac{1}{3}p(p + 1)(2p + 1)n_6, \quad (17)$$

$$6n_1^3 - 3n_2^3 - 3n_3^3 + 2n_4^3 - n_5^3 = (2p + 1)n_6^3, \quad (18)$$

$$6n_1 - 3n_2 - 3n_3 + 2n_4 - n_5 = (2p + 1)n_6. \quad (19)$$

For $p \neq 0$ and $3n_1 + n_4 \neq 0$, these imply

$$\frac{4n_6}{3n_1 + n_4} = \frac{6}{p(p + 1)(2p + 1)} = \frac{3}{2p + 1} = \left(\frac{3}{2p + 1}\right)^{1/3}. \quad (20)$$

This determines $p = 1$ (or $p = -2$ which is the same as $p = 1$ with $n_6 \rightarrow -n_6$ and $\Sigma_R \rightarrow \Sigma_L$.) In other words, the solution I have found is nontrivial and unique.

5. NuTeV Discrepancy

Since the gauge boson X couples to quarks and leptons according to Eqs. (7) and (8), it may have measurable effects in precision electroweak data, such as the deep inelastic scattering of ν_μ and $\bar{\nu}_\mu$ on nucleons in the NuTeV experiment [16]. A discrepancy has been reported in the value of the effective $\sin^2 \theta_W$, i.e. $0.2277 \pm 0.0013 \pm 0.0009$ versus the expected 0.2227 ± 0.00037 of the Standard Model. To see how this may be explained in the $U(1)_X$ model, consider the effective Hamiltonian of neutrino interactions:

$$\mathcal{H}_{int} = \frac{G_F}{\sqrt{2}} \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu [\epsilon_L^q \bar{q} \gamma_\mu (1 - \gamma_5) q + \epsilon_R^q \bar{q} \gamma_\mu (1 + \gamma_5) q]. \quad (21)$$

Assume no $Z - X$ mixing (so that the precision Z -pole measurements are not affected), then

$$\epsilon_L^u = \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W + n_1 \zeta, \quad (22)$$

$$\epsilon_L^d = -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W + n_1 \zeta, \quad (23)$$

$$\epsilon_R^u = -\frac{2}{3} \sin^2 \theta_W + n_2 \zeta, \quad (24)$$

$$\epsilon_R^d = \frac{1}{3} \sin^2 \theta_W + n_3 \zeta, \quad (25)$$

where

$$\zeta = 2n_4 \left(\frac{g_X^2}{M_X^2} \right) \left(\frac{M_Z^2}{g_Z^2} \right). \quad (26)$$

The NuTeV results versus the Standard Model predictions are:

$$(\epsilon_L^u)^2 + (\epsilon_L^d)^2 = 0.3005 \pm 0.0014 \quad \text{versus} \quad 0.3042, \quad (27)$$

$$(\epsilon_R^u)^2 + (\epsilon_R^d)^2 = 0.0310 \pm 0.0011 \quad \text{versus} \quad 0.0301. \quad (28)$$

Consider $n_1 = 1$, $n_4 = 4/3$, then $n_2 = 3/4$ and $n_3 = 5/4$. The discrepancies in the above are thus

$$\Delta_L = -\frac{2}{3} \sin^2 \theta_W \zeta + 2\zeta^2, \quad (29)$$

$$\Delta_R = -\frac{1}{6} \sin^2 \theta_W \zeta + \frac{17}{8} \zeta^2. \quad (30)$$

Let $\zeta = \sin^2 \theta_W / 6$, then a very good fit is obtained, i.e.

$$\Delta_L = -0.0028 \quad \text{versus} \quad -0.0037 \pm 0.0014, \quad (31)$$

$$\Delta_R = +0.0016 \quad \text{versus} \quad +0.0009 \pm 0.0011. \quad (32)$$

This choice also implies that $G_X/G_F = \sin^2 \theta_W / 16 = 0.014$. Note that this solution assumes that X couples to μ and the u and d quarks. If the same couplings were used for the electron, then the constraints from atomic parity violation would be grossly violated.

6. New U(1) Gauge Extension of the Supersymmetric Standard Model

In extending the Minimal Standard Model to include supersymmetry, one old problem and two new problems have to be faced. The old problem is neutrino mass. This may be solved again by adding heavy singlet neutral superfields N^c (analog of N_R). The two new problems are rapid proton decay and the value of the μ term. It has been shown recently [2] that a nontrivial U(1) gauge extension exists which cures all three problems.

Consider the following superfields with their $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_X$ assignments:

$$Q = (u, d) \sim (3, 2, 1/6; n_1), \quad u^c \sim (3^*, 1, -2/3; n_2), \quad (33)$$

$$d^c \sim (3^*, 1, 1/3; n_3), \quad L = (\nu, e) \sim (1, 2, -1/2; n_4), \quad (34)$$

$$e^c \sim (1, 1, 1; n_5), \quad N^c \sim (1, 1, 0; n_6), \quad (35)$$

$$\phi_1 \sim (1, 2, -1/2; -n_1 - n_3), \quad \phi_2 \sim (1, 2, 1/2; -n_1 - n_2). \quad (36)$$

Without $U(1)_X$, the trilinear terms LLe^c , LQd^c , and $u^cd^cd^c$ are allowed in the superpotential, thus causing rapid proton decay. The usual solution is to impose R parity, i.e. $R \equiv (-1)^{3B+L+2j}$, to forbid these terms. Even so, dimension-5 terms such as $\tilde{q}\tilde{q}ql$ are still allowed and may well be too big. Secondly, the term $\mu\phi_1\phi_2$ is allowed by supersymmetry, so there is no understanding of why μ should be of the order 1 TeV and not some very large mass such as the Planck scale or the string scale.

With $U(1)_X$, the μ problem is solved by replacing it by a singlet neutral superfield

$$\chi \sim (1, 1, 0; 2n_1 + n_2 + n_3), \quad (37)$$

where $2n_1 + n_2 + n_3 \neq 0$. The subsequent spontaneous breaking of $U(1)_X$ at the TeV scale generates the desirable value of the effective μ parameter. The solution of the proton decay problem now depends on finding an anomaly-free set of n_i 's which also forbids the undesirable terms mentioned above. With the superfield content as it is, there is no solution. However, if 2 sets of

$$U \sim (3, 1, 2/3; n_7), \quad U^c \sim (3^*, 1, -2/3; n_8), \quad (38)$$

and 1 set of

$$D \sim (3, 1, -1/3; n_7), \quad D^c \sim (3^*, 1, 1/3; n_8) \quad (39)$$

are added, a solution again appears. This will involve the remarkable exact factorization of the sum of 11 cubic terms as shown below.

There are 8 n_i 's, but 3 conditions are imposed:

$$n_1 + n_3 = n_4 + n_5, \quad n_1 + n_2 = n_4 + n_6, \quad n_7 + n_8 = -2n_1 - n_2 - n_3. \quad (40)$$

These allow ϕ_1 to give mass to the d quarks and the charged leptons, ϕ_2 to give mass to the u quarks and the neutrinos (i.e. νN^c), and χ to give mass to the U and D quarks. Consider now the anomaly-free conditions linear in X :

$$X[SU(3)]^2 : 2n_1 + n_2 + n_3 + n_7 + n_8 = 0, \quad (41)$$

$$X[SU(2)]^2 : n_2 + n_3 = 7n_1 + 3n_4, \quad (42)$$

$$X[Y]^2 : n_2 + n_3 = 7n_1 + 3n_4. \quad (43)$$

The first condition is automatically satisfied, while the other two conditions are identical and eliminate just one additional n_i . Let the remaining 4 independent n_i 's be n_1, n_2, n_4 , and n_7 , then $X^2[Y]$ implies

$$\begin{aligned} & 3n_1^2 - 2n_2^2 + n_3^2 - n_4^2 + n_5^2 + 3n_7^2 - 3n_8^2 - (n_1 + n_3)^2 + (n_1 + n_2)^2 \\ & = 6(3n_1 + n_4)(2n_1 - 4n_2 - 3n_7) = 0. \end{aligned} \quad (44)$$

The condition $3n_1 + n_4 = 0$ contradicts $2n_1 + n_2 + n_3 \neq 0$ and must be discarded. Thus $2n_1 - 4n_2 - 3n_7 = 0$ is required and only 3 independent n_i 's are left, which are chosen finally to be n_1, n_4 , and n_6 . The most nontrivial condition, i.e. X^3 , is then

$$\begin{aligned} & 18n_1^3 + 9n_2^3 + 9n_3^3 + 6n_4^3 + 3n_5^3 + 3n_6^3 + 9n_7^3 + 9n_8^3 \\ & - 2(n_1 + n_3)^3 - 2(n_1 + n_2)^3 + (2n_1 + n_2 + n_3)^3 = 0. \end{aligned} \quad (45)$$

Amazingly, this exactly factorizes to

$$-36(3n_1 + n_4)(9n_1 + n_4 - 2n_6)(6n_1 - n_4 - n_6) = 0. \quad (46)$$

Whereas the first factor cannot be zero, either of the other two can be chosen to be zero, and two solutions have been found. They are summarized in Table 1.

Table 1: Solutions (A) and (B) where $n_i = an_1 + bn_4$.

	(A)		(B)	
	a	b	a	b
n_2	7/2	3/2	5	0
n_3	7/2	3/2	2	3
n_5	9/2	1/2	3	2
n_6	9/2	1/2	6	-1
n_7	-4	-2	-6	0
n_8	-5	-1	-3	-3
$-n_1 - n_3$	-9/2	-3/2	-3	-3
$-n_1 - n_2$	-9/2	-3/2	-6	0
$2n_1 + n_2 + n_3$	9	3	9	3

Table 2: Conditions on n_1 and n_4 in (A) and (B).

(A)		(B)		
c	d	c	d	$cn_1 + dn_4 \neq 0$ forbids
3	1	3	1	μ term
9	5	3	4	L violation
7	3	3	2	B violation
1	1	1	3	U^c as diquark
1	1	1	0	D^c as diquark
1	0	5	-1	U as leptoquark
1	0	1	1	D as leptoquark
13	1	4	3	U^c, D^c as semiquarks

In Table 2 the various conditions of forbidding B and L violation, etc. in the superpotential are listed. The condition $3n_1 + n_4 \neq 0$ which forbids the μ term also forbids the higher-dimensional terms $QQQL$ and $u^c u^c d^c e^c$, which are allowed by R parity. Thus proton decay is much more effectively suppressed in this case. Note also that solutions (A) and (B) are the same for $n_1 = n_4$.

7. Naturally Small Dirac Neutrino Masses

The sum of X charges is

$$3(6n_1 + 3n_2 + 3n_3 + 2n_4 + n_5 + n_6) + 3(3n_7 + 3n_8) + 2(-n_1 - n_3) + 2(-n_1 - n_2) + (2n_1 + n_2 + n_3) = 6(3n_1 + n_4) \neq 0. \quad (47)$$

To get rid of this mixed gravitational-gauge anomaly, add the following singlet superfields in units of $3n_1 + n_4$: one with charge 3, three (S^c) with charge -2 , and three (N) with charge -1 . Then since

$$1(3) + 3(-2) + 3(-1) = -6, \quad 1(3)^3 + 3(-2)^3 + 3(-1)^3 = 0, \quad (48)$$

the mixed gravitational-gauge anomaly is canceled without affecting Eq. (45).

If now $n_6 = 3n_1 + n_4$ is assumed, then there are 3 copies each of $N^c(n_6)$, $N(-n_6)$, and $S^c(-2n_6)$. Thus NN^c forms a large invariant mass M , $NS^c\chi$ implies a mass m_2 proportional to $\langle\chi\rangle$, and $\nu N^c\phi_2^0$ implies a mass m_1 proportional to $\langle\phi_2^0\rangle$. The 12×12 mass matrix spanning (ν, S^c, N, N^c) is then of the form

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & 0 & m_1 \\ 0 & 0 & m_2 & 0 \\ 0 & m_2 & 0 & M \\ m_1 & 0 & M & 0 \end{pmatrix}. \quad (49)$$

With $m_1 \sim 10^2$ GeV, $m_2 \sim 10^3$ GeV, and $M \sim 10^{16}$ GeV, this implies that neutrinos have naturally small Dirac masses of order

$$m_\nu = \frac{m_1 m_2}{M} \sim 10^{-2} \text{ eV}. \quad (50)$$

8. Nearly Degenerate Majorana Neutrino Masses

I now switch gear and consider the derivation of the observed pattern of solar [3] and atmospheric [4] neutrino oscillations in the context of nearly degenerate Majorana neutrino masses. This is based [17] on the non-Abelian discrete symmetry A_4 (i.e. the symmetry group of a regular tetrahedron, the simplest of the 5 perfect geometric solids). It will be shown [18] that radiative corrections automatically generate the desired neutrino mass matrix and if these come from softly broken supersymmetry, then the effective mass measured in neutrinoless double beta decay [5] should not be much smaller than about 0.4 eV.

Suppose that at some high energy scale, the charged lepton mass matrix and the Majorana neutrino mass matrix are such that after diagonalizing the former, i.e.

$$\mathcal{M}_l = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad (51)$$

the latter is of the form

$$\mathcal{M}_\nu = \begin{pmatrix} m_0 & 0 & 0 \\ 0 & 0 & m_0 \\ 0 & m_0 & 0 \end{pmatrix}. \quad (52)$$

From the high scale to the electroweak scale, one-loop radiative corrections will change \mathcal{M}_ν as follows:

$$(\mathcal{M}_\nu)_{ij} \rightarrow (\mathcal{M}_\nu)_{ij} + R_{ik}(\mathcal{M}_\nu)_{kj} + (\mathcal{M}_\nu)_{ik}R_{kj}, \quad (53)$$

where the radiative correction matrix is assumed to be of the most general form, i.e.

$$R = \begin{pmatrix} r_{ee} & r_{e\mu} & r_{e\tau} \\ r_{e\mu}^* & r_{\mu\mu} & r_{\mu\tau} \\ r_{e\tau}^* & r_{\mu\tau}^* & r_{\tau\tau} \end{pmatrix}. \quad (54)$$

Thus the observed neutrino mass matrix is given by

$$\mathcal{M}_\nu = m_0 \begin{pmatrix} 1 + 2r_{ee} & r_{e\tau} + r_{e\mu}^* & r_{e\mu} + r_{e\tau}^* \\ r_{e\mu}^* + r_{e\tau} & 2r_{\mu\tau} & 1 + r_{\mu\mu} + r_{\tau\tau} \\ r_{e\tau}^* + r_{e\mu} & 1 + r_{\mu\mu} + r_{\tau\tau} & 2r_{\mu\tau}^* \end{pmatrix}. \quad (55)$$

Consider first the case where all the parameters are real. Redefine them as follows:

$$\delta_0 \equiv r_{\mu\mu} + r_{\tau\tau} - 2r_{\mu\tau}, \quad (56)$$

$$\delta \equiv 2r_{\mu\tau}, \quad (57)$$

$$\delta' \equiv r_{ee} - \frac{1}{2}r_{\mu\mu} - \frac{1}{2}r_{\tau\tau} - r_{\mu\tau}, \quad (58)$$

$$\delta'' \equiv r_{e\mu} + r_{e\tau}. \quad (59)$$

Then the neutrino mass matrix becomes

$$\mathcal{M}_\nu = m_0 \begin{pmatrix} 1 + \delta_0 + 2\delta + 2\delta' & \delta'' & \delta'' \\ \delta'' & \delta & 1 + \delta_0 + \delta \\ \delta'' & 1 + \delta_0 + \delta & \delta \end{pmatrix}. \quad (60)$$

This matrix is *exactly* diagonalized with the eigenvalues

$$m_1 = m_0(1 + 2\delta + \delta' - \sqrt{\delta'^2 + 2\delta''^2}), \quad (61)$$

$$m_2 = m_0(1 + 2\delta + \delta' + \sqrt{\delta'^2 + 2\delta''^2}), \quad (62)$$

$$m_3 = -m_0, \quad (63)$$

where δ_0 has been set equal to zero by a trivial rescaling of the other parameters, and the neutrino mixing matrix is given by

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta / \sqrt{2} & \cos \theta / \sqrt{2} & -1/\sqrt{2} \\ \sin \theta / \sqrt{2} & \cos \theta / \sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad (64)$$

where

$$\tan \theta = \frac{\sqrt{2}\delta''}{\sqrt{\delta'^2 + 2\delta''^2} - \delta'}, \quad (\delta' < 0). \quad (65)$$

This is exactly the right description of present data on solar and atmospheric neutrino oscillations, with $\sin^2 2\theta_{atm} = 1$ and $\tan^2 \theta_{sol} = 0.38$ if $|\delta''/\delta'| = \sqrt{2}$ for example. It also predicts that the common mass of the three neutrinos, i.e. m_0 , is what is measured in neutrinoless double beta decay.

9. Discrete A_4 Symmetry

The successful derivation of Eq. (64) depends on having Eqs. (51) and (52). To be sensible theoretically, they should be maintained by a symmetry. At first sight, it appears impossible that there can be a symmetry which allows them to coexist. Here is where the non-Abelian discrete symmetry A_4 comes into play. The key is that A_4 has three inequivalent one-dimensional representations $\underline{1}$, $\underline{1}'$, $\underline{1}''$, and one three-dimensional representation $\underline{3}$, with the decomposition

$$\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3} + \underline{3}. \quad (66)$$

This allows the following natural assignments of quarks and leptons:

$$(u_i, d_i)_L, (\nu_i, e_i)_L \sim \underline{3}, \quad (67)$$

$$u_{1R}, d_{1R}, e_{1R} \sim \underline{1}, \quad (68)$$

$$u_{2R}, d_{2R}, e_{2R} \sim \underline{1}', \quad (69)$$

$$u_{3R}, d_{3R}, e_{3R} \sim \underline{1}''. \quad (70)$$

Heavy fermion singlets are then added:

$$U_{iL(R)}, D_{iL(R)}, E_{iL(R)}, N_{iR} \sim \underline{3}, \quad (71)$$

together with the usual Higgs doublet and new heavy singlets:

$$(\phi^+, \phi^0) \sim \underline{1}, \quad \chi_i^0 \sim \underline{3}. \quad (72)$$

With this structure, charged leptons acquire an effective Yukawa coupling matrix $\bar{e}_{iL} e_{jR} \phi^0$ which has 3 arbitrary eigenvalues (because of the 3 independent couplings to the 3 inequivalent one-dimensional representations) and for the case of equal vacuum expectation values of χ_i , i.e.

$$\langle \chi_1 \rangle = \langle \chi_2 \rangle = \langle \chi_3 \rangle = u, \quad (73)$$

the unitary transformation U_L which diagonalizes \mathcal{M}_l is given by

$$U_L = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad (74)$$

where $\omega = e^{2\pi i/3}$. This implies that the effective neutrino mass operator, i.e. $\nu_i \nu_j \phi^0 \phi^0$, is proportional to

$$U_L^T U_L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (75)$$

exactly as desired [17, 18].

10. Softly Broken Supersymmetry

To derive Eq. (75), the validity of Eq. (73) has to be proved. This is naturally accomplished in the context of supersymmetry. Let $\hat{\chi}_i$ be superfields, then its superpotential is given by

$$\hat{W} = \frac{1}{2} M_\chi (\hat{\chi}_1 \hat{\chi}_1 + \hat{\chi}_2 \hat{\chi}_2 + \hat{\chi}_3 \hat{\chi}_3) + h_\chi \hat{\chi}_1 \hat{\chi}_2 \hat{\chi}_3. \quad (76)$$

Note that the h_χ term is invariant under A_4 , a property not found in $SU(2)$ or $SU(3)$. The resulting scalar potential is

$$V = |M_\chi + h_\chi \chi_2 \chi_3|^2 + |M_\chi \chi_2 + h_\chi \chi_3 \chi_1|^2 + |M_\chi \chi_3 + h_\chi \chi_1 \chi_2|^2. \quad (77)$$

Thus a supersymmetric vacuum ($V = 0$) exists for

$$\langle \chi_1 \rangle = \langle \chi_2 \rangle = \langle \chi_3 \rangle = u = -M_\chi / h_\chi, \quad (78)$$

proving Eq. (73), with the important additional result that the spontaneous breaking of A_4 at the high scale u does not break the supersymmetry.

To generate the proper radiative corrections which will result in a realistic Majorana neutrino mass matrix, A_4 is assumed broken also by the soft supersymmetry breaking terms. In particular, the mass-squared matrix of the left sleptons will be assumed to be arbitrary. This allows $r_{\mu\tau}$ to be nonzero through $\tilde{\mu}_L - \tilde{\tau}_L$ mixing, from which the parameter δ may be evaluated. For illustration, using the approximation that $\tilde{m}_1^2 \gg \mu^2 \gg M_{1,2}^2 = \tilde{m}_2^2$, where μ is the Higgsino mass and $M_{1,2}$ are gaugino masses, I find

$$\delta = \frac{\sin \theta \cos \theta}{16\pi^2} \left[(3g_2^2 - g_1^2) \ln \frac{\tilde{m}_1^2}{\mu^2} - \frac{1}{4} (3g_2^2 + g_1^2) \left(\ln \frac{\tilde{m}_1^2}{\tilde{m}_2^2} - \frac{1}{2} \right) \right]. \quad (79)$$

Using $\Delta m_{32}^2 = 2.5 \times 10^{-3} \text{ eV}^2$ from the atmospheric neutrino data, this implies that

$$\left[\ln \frac{\tilde{m}_1^2}{\mu^2} - 0.3 \left(\ln \frac{\tilde{m}_1^2}{\tilde{m}_2^2} - \frac{1}{2} \right) \right] \sin \theta \cos \theta \simeq 0.535 \left(\frac{0.4 \text{ eV}}{m_0} \right)^2. \quad (80)$$

To the extent that the factor on the left cannot be much greater than unity, this means that m_0 cannot be much smaller than about 0.4 eV [5].

11. Nonzero U_{e3} and CP Violation

The matrix of Eq. (55) has complex phases. It can easily be shown that only one phase remains after all possible redefinitions. As a result, the most general Majorana neutrino mass matrix derivable from A_4 is actually of the form

$$\mathcal{M}_\nu = m_0 \begin{pmatrix} 1 + 2\delta + 2\delta' & \delta'' & \delta''^* \\ \delta'' & \delta & 1 + \delta \\ \delta''^* & 1 + \delta & \delta \end{pmatrix}, \quad (81)$$

where only δ'' is complex. In this case, U_{e3} becomes nonzero, i.e.

$$U_{e3} \simeq \frac{i \text{Im} \delta''}{\sqrt{2} \delta}. \quad (82)$$

Note the important result that U_{e3} is purely imaginary. Thus CP violation in neutrino oscillations is predicted to be maximal, which is desirable for future long-baseline neutrino experiments.

In the presence of $\text{Im} \delta''$, the previous expressions are still approximately valid with the replacement of δ' by $\delta' + (\text{Im} \delta'')^2 / 2\delta$ and of δ'' by $\text{Re} \delta''$. There is also the relationship

$$\left[\frac{\Delta m_{12}^2}{\Delta m_{32}^2} \right]^2 \simeq \left[\frac{\delta'}{\delta} + |U_{e3}|^2 \right]^2 + \left[\frac{\text{Re} \delta''}{\delta} \right]^2. \quad (83)$$

Using $\Delta m_{12}^2 \simeq 5 \times 10^{-5} \text{ eV}^2$ from solar neutrino data and $|U_{e3}| < 0.16$ from reactor neutrino data [19], I find

$$\text{Im} \delta'' < 8.8 \times 10^{-4} (0.4 \text{ eV} / m_0)^2, \quad (84)$$

$$\text{Re} \delta'' < 7.8 \times 10^{-5} (0.4 \text{ eV} / m_0)^2. \quad (85)$$

12. Conclusions

There are undoubtedly oases in the desert beyond the Standard Model. Two first examples of nontrivial U(1) gauge extensions of the Standard Model and its supersymmetric generalization have been discovered. In the first case [1], a heavy lepton triplet (Σ^+ , Σ^0 , Σ^-) per family is added to the Standard Model as the anchor for a naturally small seesaw Majorana neutrino mass (instead of using the canonical singlet N_R). This allows for the remarkable existence of a new U(1) gauge symmetry, with possible phenomenological consequences already at the electroweak scale, such as the NuTeV discrepancy. In the second case [2], with the addition of new superfields to the Supersymmetric Standard Model, a new U(1) gauge symmetry is again discovered which forbids proton decay, explains the size of the μ term, and allows for naturally small Dirac neutrino masses. The existence of this new gauge symmetry involves the highly nontrivial exact factorization of the sum of 11 cubic terms.

A third possible oasis is that of an underlying A_4 symmetry at some high energy scale, which allows the observed Majorana neutrino mass matrix to be derived from radiative corrections. It has been shown [18] that this automatically leads to $\sin^2 2\theta_{atm} = 1$ and a large (but not maximal) solar mixing angle. Using neutrino oscillation data, and assuming radiative corrections from soft supersymmetry breaking, the effective mass measured in neutrinoless double beta decay is predicted to be not much less than 0.4 eV.

Acknowledgements

I thank Hans Klapdor, Juha Peltoniemi, and the other organizers of Beyond 2002 for their great hospitality at Oulu. This work was supported in part by the U. S. Department of Energy under Grant No. DE-FG03-94ER40837.

References

- [1] E. Ma, Mod. Phys. Lett. A 17, 535 (2002); E. Ma and D. P. Roy, hep-ph/0206150.
- [2] E. Ma, Phys. Rev. Lett. 89, 041801 (2002).
- [3] Q. R. Ahmad *et al.*, SNO Collaboration, Phys. Rev. Lett. 89, 011301, 011302 (2002); S. Fukuda *et al.*, Super-Kamiokande Collaboration, Phys. Rev. Lett. 86, 5656 (2001) and references therein.
- [4] S. Fukuda *et al.*, Super-Kamiokande Collaboration, Phys. Rev. Lett. 85, 3999 (2000) and references therein.
- [5] H. V. Klapdor-Kleingrothaus *et al.*, Mod. Phys. Lett. A 16, 2409 (2001).
- [6] S. L. Adler, Phys. Rev. 177, 2426 (1969); J. S. Bell and R. Jackiw, Nuovo Cimento A 60, 47 (1969); W. A. Bardeen, Phys. Rev. 184, 1848 (1969).
- [7] C. Bouchiat, J. Iliopoulos, and P. Meyer, Phys. Lett. B 38, 579 (1972).
- [8] S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).
- [9] X. G. He, G. C. Joshi, H. Lew, and R. R. Volkas, Phys. Rev. D 43, 22 (1991); 44, 2118 (1991).
- [10] E. Ma, Phys. Lett. B 433, 74 (1998); E. Ma and U. Sarkar, Phys. Lett. B 439, 95 (1998); E. Ma and D. P. Roy, Phys. Rev. D 58, 095005 (1998); E. Ma, D. P. Roy, and U. Sarkar, Phys. Lett. B 444, 391 (1998).
- [11] E. Ma and D. P. Roy, Phys. Rev. D 59, 097702 (1999).
- [12] R. E. Marshak and R. N. Mohapatra, Phys. Lett. B 91, 222 (1980).
- [13] R. Foot, H. Lew, X.-G. He, and G. C. Joshi, Z. Phys. C 44, 441 (1989).
- [14] E. Ma, Phys. Rev. Lett. 81, 1171 (1998).
- [15] R. Delbourgo and A. Salam, Phys. Lett. B 40, 381 (1972); T. Eguchi and P. G. O. Freund, Phys. Rev. Lett. 37, 1251 (1976); L. Alvarez-Gaume and E. Witten, Nucl. Phys. B 234, 269 (1984).
- [16] G. P. Zeller *et al.*, NuTeV Collaboration, Phys. Rev. Lett. 88, 091802 (2002).
- [17] E. Ma and G. Rajasekaran, Phys. Rev. D 64, 113012 (2001); E. Ma, Mod. Phys. Lett. A 17, 289 (2002); E. Ma, Mod. Phys. Lett. A 17, 627 (2002).
- [18] K. S. Babu, E. Ma, and J. W. F. Valle, hep-ph/0206292.
- [19] M. Apollonio *et al.*, Phys. Lett. B 466, 415 (1999); F. Boehm *et al.*, Phys. Rev. D 64, 112001 (2001).